9-1/9-2 Vectors

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$\vec{AB} = B - A$

Ex1. Given A(4, -7) and B(2,-2). Find:

$\langle 2-4, -2--7 \rangle$

a.) $\overrightarrow{AB}$ $= \boxed{\langle -2, 5 \rangle}$

b.) The magnitude of $\overrightarrow{AB}$

$\sqrt{(-2)^2 + 5^2} = \sqrt{29}$

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Ex1. Given A(4, -7) and B(2,-2). Find:

c.) The direc?on with respect to the posi?ve x-axis. $\langle -2, 5 \rangle$

$\tan^{-1}\left(-\frac{5}{2}\right) = -68.2°$
$+180$
$111.8°$

d.) The vector $\overrightarrow{AB}$ in polar form.

$(\sqrt{29}; 111.8°)$

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Conver?ng Between Polar and Component Form

Component to Polar	**Polar to Component**
Given vector $\langle x, y \rangle$ in component form, then $\lvert\langle x, y\rangle\rvert = \sqrt{x^2 + y^2}$; $\theta = \tan^{-1}\left(\frac{y}{x}\right)$	Given vector $(r; \theta)$ in polar form, then $x = r\cos\theta$; $y = r\sin\theta$

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A unit vector is a vector which is 1 unit long. Reduced in length (or possibly lengthened) to a magnitude of 1 while preserving the director of the vector.

Unit vector for $\vec{v} = \frac{\vec{v}}{|\vec{v}|}$

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Ex1. Given A(4, -7) and B(2,-2). Find:

e.) The unit vector for $\overrightarrow{AB}$

$\vec{v} = \langle -2, 5 \rangle$

$\frac{\vec{v}}{|\vec{v}|} = \frac{\langle -2, 5 \rangle}{\sqrt{29}} = \boxed{\left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle}$

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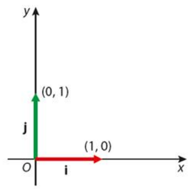
Base Vectors in the Coordinate Plane



i = unit vector in the direction of the positive x-axis

j = unit vector in the direction of the positive y-axis.

Hence the vector $\langle -2, 5\rangle$ can also be written -2***i*** + 5***j***

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Vector Format

Row Vector $\langle 3,-2\rangle$ or $(3,-2)$

Column Vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Unit Vector notation 3i – 2j

Polar Form $(\sqrt{13};146°)$

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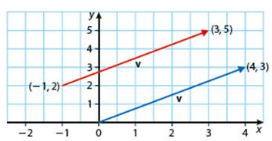
Two vectors $\vec{u}$ and $\vec{v}$ are equal if they have the same magnitude and direction.



The negative of vector $\vec{u}$ denoted $-\vec{u}$, is a vector with the same magnitude but the opposite direction.

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Vector Addition – Geometric Representation

Tip-to-Tail Method

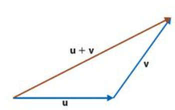

Figure 9.7

Using this method, you place the tail of the second vector against the tip of the first vector. The vector sum is the vector that connects the tail of the first vector to the tip of the second vector.

Parallelogram Method

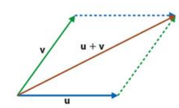

Figure 9.8

Using this method, you place both tails at the same location and the vector sum is the diagonal of the parallelogram.

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Vector Subtraction

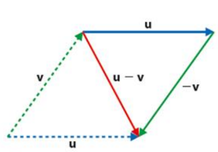

Figure 9.9

The difference of two vectors is the other diagonal of the parallelogram. Think of it this way:

$\vec{u} - \vec{v} \Rightarrow \vec{u} + \left(-\vec{v}\right)$

What would be $\vec{v} - \vec{u}$? $-\vec{v} - \vec{u}$? $-\vec{u} - \vec{v}$?

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Vector Addition – Algebraic Representation

Ex2. Let $\vec{u} = \begin{pmatrix} 1 \\ -6 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$. Find:

$\vec{u} + \vec{v} =$ $\langle -2, -11\rangle$ $\begin{pmatrix} -2 \\ -11 \end{pmatrix}$

$\vec{u} - \vec{v} =$ $\langle 4, -1\rangle$ $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

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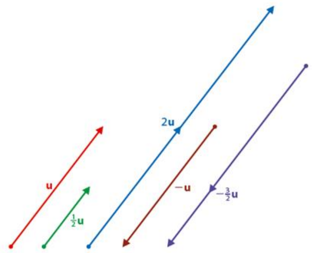
Scalar Multiplication

Changes the length of a vector while preserving the direction.



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Ex3. Let $\vec{u} = \begin{pmatrix} 1 \\ -6 \end{pmatrix}$ and $\vec{v} = 2\mathbf{i} + 3\mathbf{j}$. Find:

a.) $7\vec{u}$ $\begin{pmatrix} 7 \\ -42 \end{pmatrix}$

b.) $-2\vec{u}$ $\begin{pmatrix} -2 \\ 12 \end{pmatrix}$

c.) $4\vec{v}$ $\begin{pmatrix} 8 \\ 12 \end{pmatrix}$

d.) $2\vec{u} + 3\vec{v}$ $\begin{pmatrix} 2 \\ -12 \end{pmatrix} + \begin{pmatrix} 6 \\ 9 \end{pmatrix} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$

e.) $-\frac{1}{2}\vec{v}$ $-\mathbf{i} - \frac{3}{2}\mathbf{j}$

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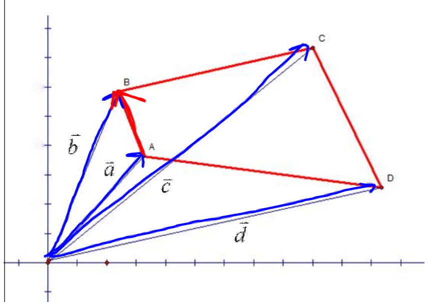
Ex4. ABCD is a quadrilateral with vertices that have position vectors a, b, c, and d respectively.



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a.) Express each of the following in terms of $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$. Find:

a.) $\overline{AB} = \vec{b} - \vec{a}$

b.) $\overline{CD} = \vec{d} - \vec{c}$

c.) $\overline{BC} = \vec{c} - \vec{b}$

d.) $\overline{AD} = \vec{d} - \vec{a}$

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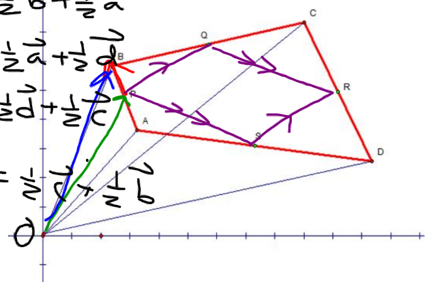
b.) P, Q, R, S are the midpoints of each each side. Express each of the following in terms of $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$.

a.) $\overline{OP} = \frac{1}{2}\vec{b} + \frac{1}{2}\vec{a}$

b.) $\overline{OS} = \frac{1}{2}\vec{a} + \frac{1}{2}\vec{d}$

c.) $\overline{OR} = \frac{1}{2}\vec{d} + \frac{1}{2}\vec{c}$

d.) $\overline{OQ} = \frac{1}{2}\vec{c} + \frac{1}{2}\vec{b}$



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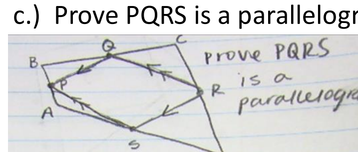
c.) Prove PQRS is a parallelogram.



$\overrightarrow{QP} = \overrightarrow{PB} + \overrightarrow{BQ} = \frac{1}{2}(\vec{b} - \vec{a}) + \frac{1}{2}(\vec{c} - \vec{b}) = \frac{1}{2}\vec{b} - \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a} + \frac{1}{2}\vec{c}$

$\overrightarrow{QP} = -\frac{1}{2}\vec{a} + \frac{1}{2}\vec{c}$

$\overrightarrow{RS} = \overrightarrow{SD} + \overrightarrow{DR} = \frac{1}{2}(\vec{d} - \vec{a}) + \frac{1}{2}(\vec{c} - \vec{d}) = \frac{1}{2}\vec{d} - \frac{1}{2}\vec{d} - \frac{1}{2}\vec{a} + \frac{1}{2}\vec{c}$

$\overrightarrow{RS} = -\frac{1}{2}\vec{a} + \frac{1}{2}\vec{c}$

$\overrightarrow{RS} = \overrightarrow{QP}$

$\overrightarrow{RQ} = \overrightarrow{QC} + \overrightarrow{CR} = \frac{1}{2}(\vec{c} - \vec{b}) + \frac{1}{2}(\vec{d} - \vec{c})$

$\overrightarrow{RQ} = -\frac{1}{2}\vec{b} + \frac{1}{2}\vec{d}$

$\overrightarrow{SP} = \overrightarrow{PA} + \overrightarrow{AS} = \frac{1}{2}(\vec{a} - \vec{b}) + \frac{1}{2}(\vec{d} - \vec{a})$

$\overrightarrow{SP} = -\frac{1}{2}\vec{b} + \frac{1}{2}\vec{d}$ $\overrightarrow{RQ} = \overrightarrow{SP}$

since $\overrightarrow{RS} = \overrightarrow{QP}$ and $\overrightarrow{RQ} = \overrightarrow{SP}$, PQRS is a parallelogram.

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